DECISIVE ANALYTICS

Solutions For Success

Ensuring Schedulability in the Weapon Target Assignment Problem

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A Systems Engineering Company Providing Analytical Solutions To Complex Problems

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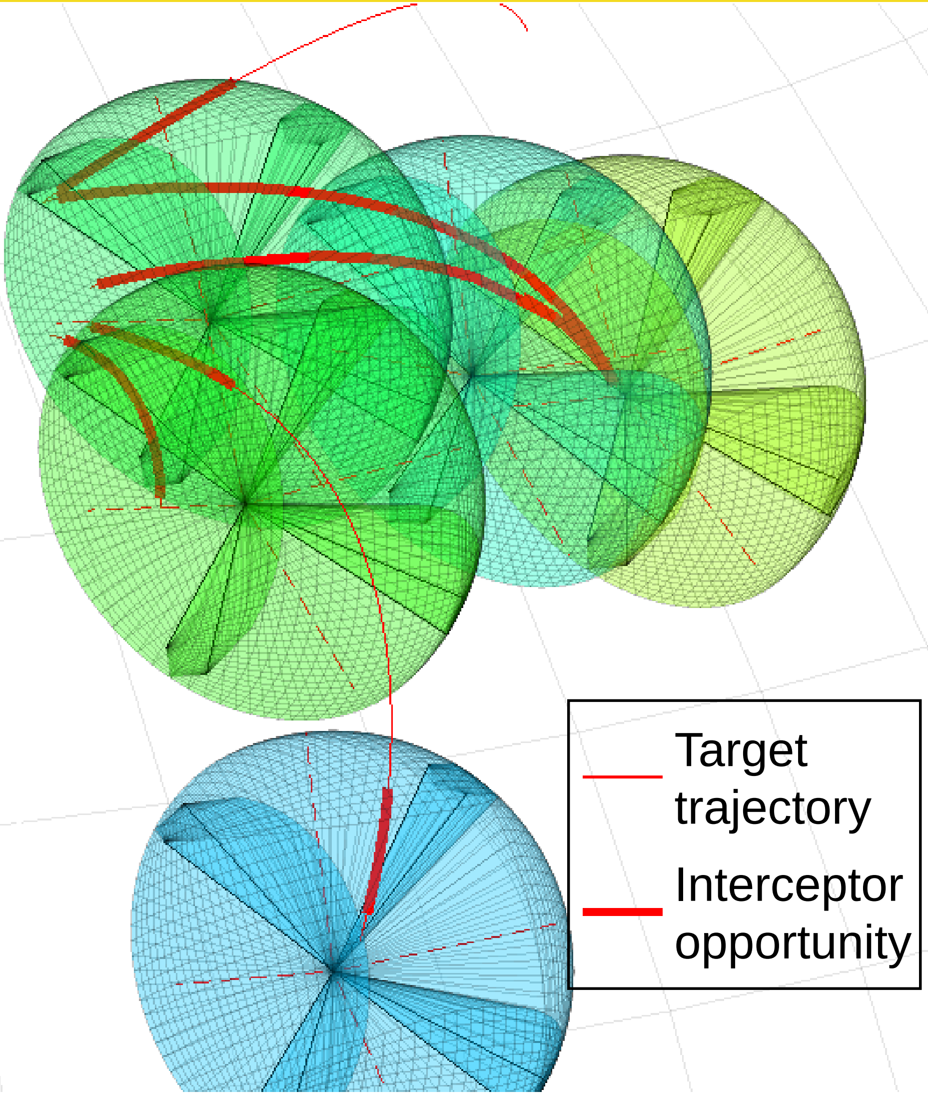
Outline

- The Weapon-Target Assignment Problem
- Deployment Related Issues
- Separability
- Conclusion

Weapon Target Assignment (WTA)

Given a set of weapon batteries and a set of incoming targets, what is the assignment of interceptors to targets that will maximize the value of targets killed?



- Difficult problem to solve optimally for large numbers of weapons/targets.
- May need to be re-solved repeatedly as battlespace evolves.
- Need good, quick solution method

Previous Work

- Maximum Marginal Return (denBroeder et al, 1958)
  - Greedy algorithm makes assignments to maximize marginal contribution
  - Optimal in some instances
- Network Flow (Castanon et al, 1987)
- Genetic Algorithms (Grant et al, 1993)

These approaches will solve the allocation problem, but do not consider the schedulability of the results

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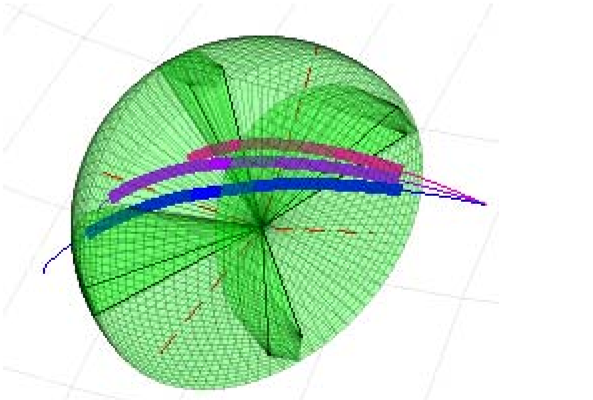
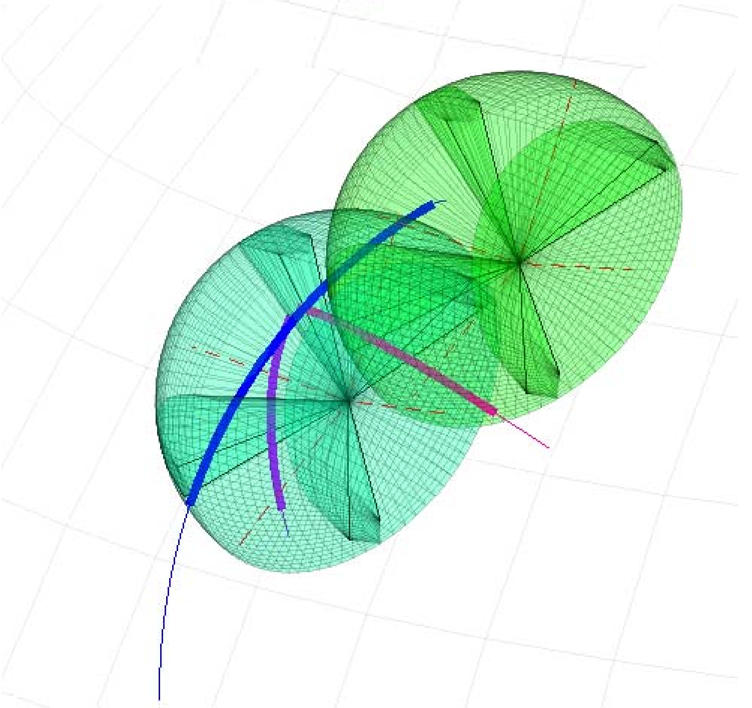
Schedulability

- Individual weapon site's scheduling system could invalidate allocation
  - Default schedule might conflict with other site's default schedule
  - Allocated more than are launchable
  - Required time between intercepts



- Benefits from coordination
  - Inter-site Shoot-Look-Shoot



Scheduling Constraints

- Inter-site coordination introduces a different set of problems
  - Inter-intercept requirements
  - Communication of kill assessment
- Other constraints include
  - Inter-launch requirements
  - Maximum number of interceptors in air at a time
  - Maximum number of distinct threats targeted

Weapon Allocation + Scheduling

Maximize expected value of threats intercepted

$$\sum_{j=1}^{n} V_j \left[1 - \prod_{i=1}^{m} \left(1 - p_{ij}\right)^{x_{ij}} \right]$$

subject to

- Inventory
- Min/max allocation limits
- Inter-launch timing
- Inter-intercept timing
- Max in the air at a time
- Max # distinct threats engaged

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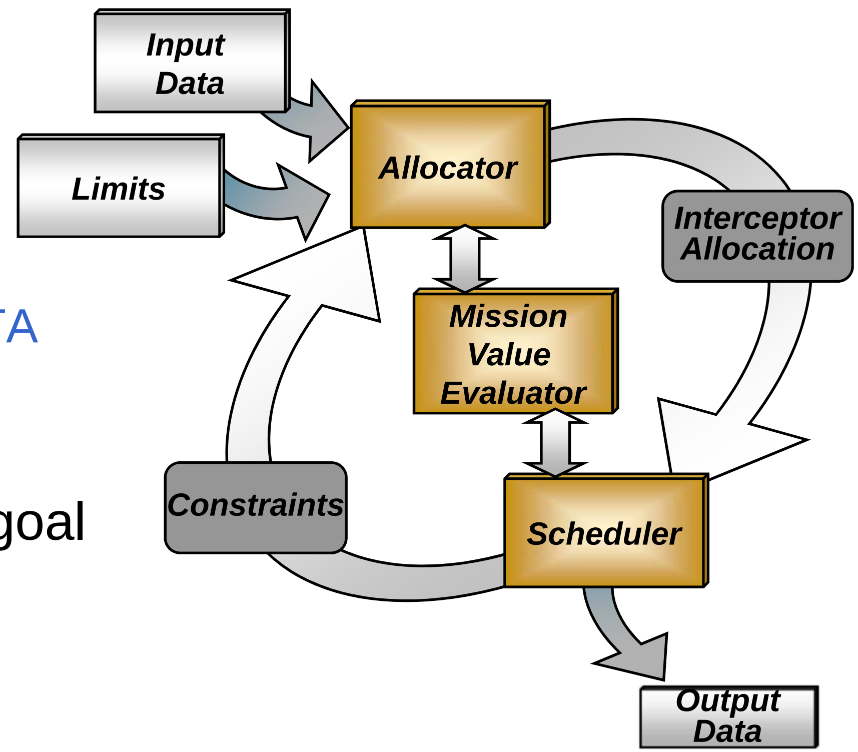
Solution Approach

- Goal: Compute a schedulable allocation of interceptors to threats within a relatively short amount of time
- Adding scheduling to the WTA problem drastically expands the problem size
- Propose problem separation
  - Solve WTA
  - Attempt to schedule this allocation
  - If not all scheduled, constrain the WTA problem and repeat
- WTA/Scheduling share objective function; both working toward same goal



Benefits

- Separability makes it possible to solve allocation and scheduling problem separately in a reasonable time
  - Implemented in Matlab, meets given run time specification
- Could implement different (related) goals for allocation/scheduling portions. For example:
  - Allocation module uses tactics (coordinated interceptor plans from multiple sites)
  - Scheduling is done interceptor by interceptor, addressing highest valued threat first
- Modular algorithm improvements
  - Replace Allocation or Scheduling section without affecting the other

Modular Development Example

- Problem: Operators want to be able to ensure all threats are engaged
- Previously had been solving allocation problem with tactics-based MMR, which tended to lead to over-engagement of high valued threats
- Solution: Implement two-pass allocation algorithm
  - First pass: Constraint programming inspired solution ensures operator’s goal is met
  - Second pass: MMR-like improvement

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Improvements & Future Work

- Allocation
  - Incorporate MHMMR to improve responsiveness to changing mission value functions
- Scheduling
  - Develop a less rudimentary heuristic
  - Incorporate time-varying success probabilities

Conclusion

- Classic WTA solutions are not concerned with the schedulability of the allocations
- Any deployed coordination system must take the scheduling of interceptors into account
- To maintain reasonable run-time, we have implemented a allocate/schedule cycle. Each part solves a small problem quickly
- This also provides a good basis for quickly incorporating improved algorithms

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Backup

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References

- denBroeder, G.G., Ellison, R.E. and Emerling, L., "On Optimum Target Assignments", *Operations Research,* Vol.7, pp. 322-326, 1959.
- Lloyd, S.P. and Witsenhausen, H.S., "Weapons Allocation is NP-Complete", Proceedings of 1986 Summer Conference on Simulation, Reno, Nevada, July 1986.

Maximum Marginal Return

- Greedy heuristic for solving the WTA problem
- MMR procedure
  - Calculate the marginal value that would result for all possible interceptor/target pairings
  - Find the maximum contribution, assign that interceptor to the target
  - Recalculate the marginal values, repeat until all available interceptors have been assigned

Graphical example assigning two interceptors:

Maximum Marginal Return

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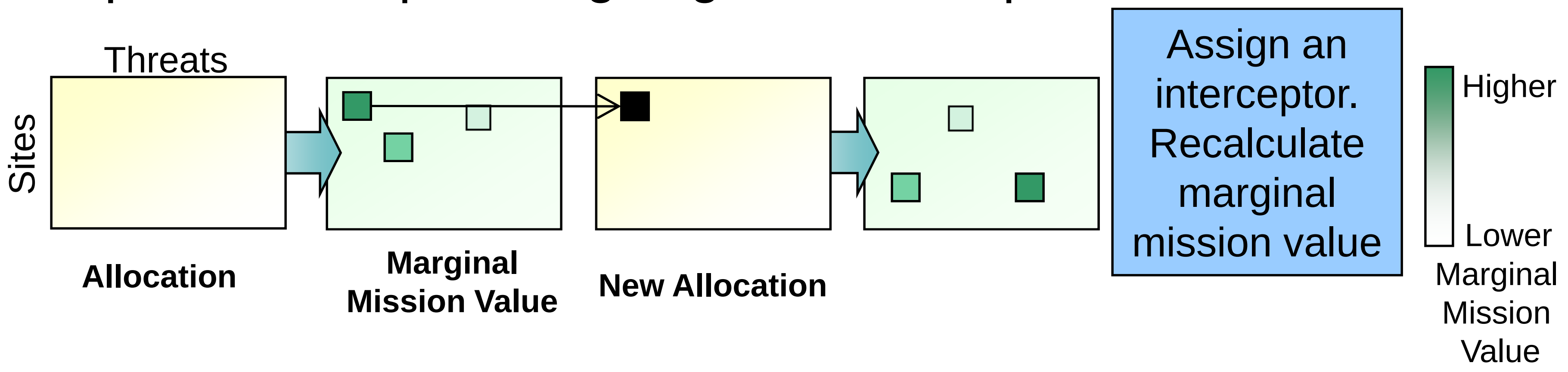
Graphical example assigning two interceptors:



Maximum Marginal Return

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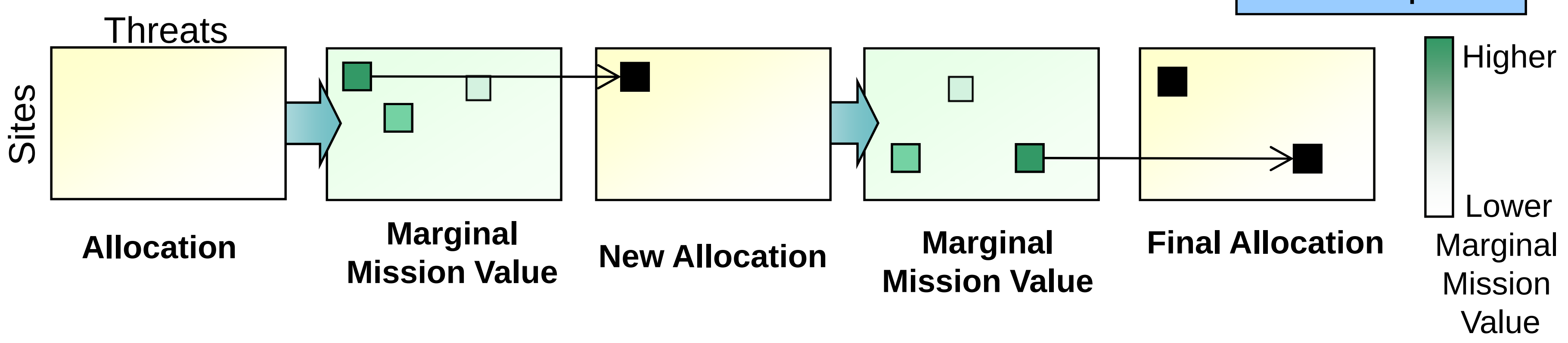
Graphical example assigning two interceptors:

Assign the second interceptor



Adapting MMR

- Pros
  - Very fast: Run-time proportional to inventory
  - Good solution: Guaranteed optimal for target-based mission value if kill probabilities are equal between sites
- Cons
  - Limited scope: Unable to solve asset-based WTA because only considers single interceptor assignments
  - Not optimal in general: Susceptible to localized maxima

Goal: Adapt MMR to make a more flexible algorithm to use when final form of mission value is uncertain.

Outline

- Maximum Marginal Return (MMR)
- Multi-Hypothesis MMR
- Results
- Conclusion

Multi-Hypothesis MMR

- Goals
  - Better -- Improve on MMR solution
  - Fast -- Maintain run-time
  - Flexible -- applicable to more possible Mission Value functions
  - Quick development of algorithm

- Enumeration provides optimal solution, but quickly becomes intractable as # targets or inventory grows
- Other methods may give optimal solution for particular Mission Value functions

MHMMR combines search properties of Enumeration and speed of MMR to provide good, fast solution for variety of objectives

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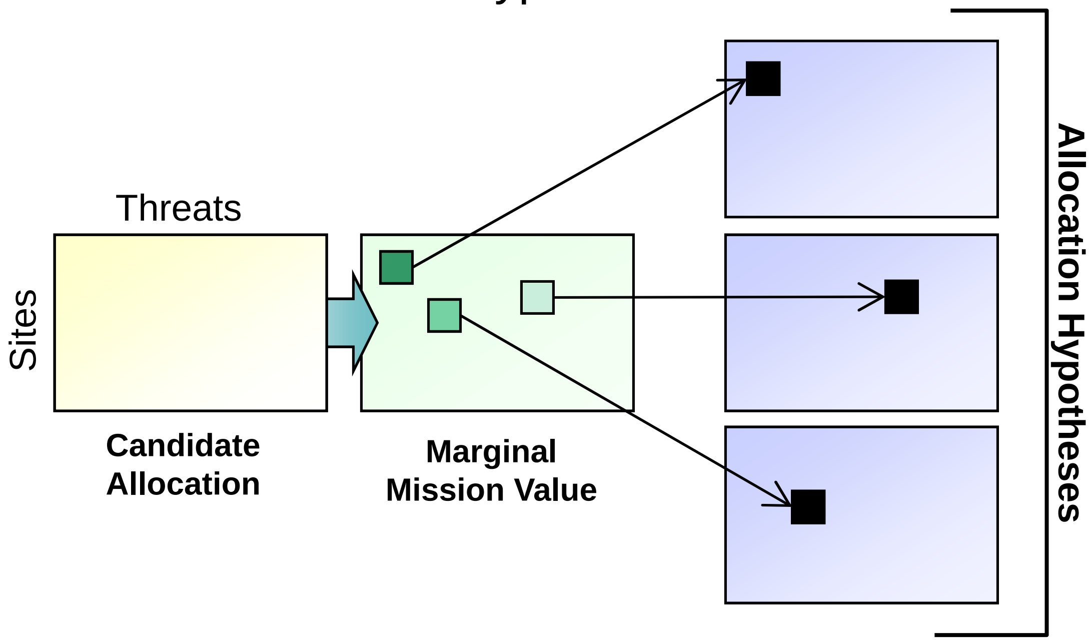
MHMMR - Branch

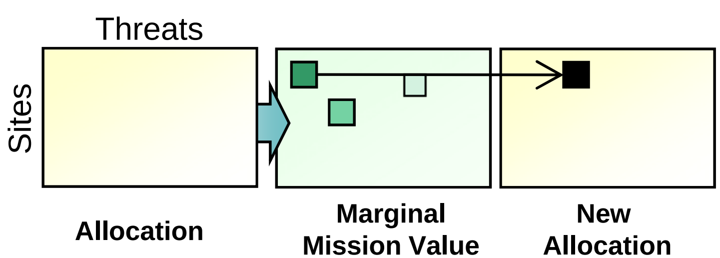
Procedure for Multi-Hypothesis MMR

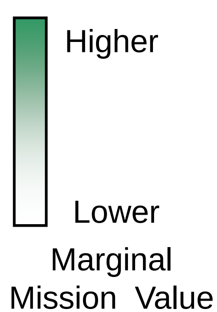


MHMMR

- Marginal mission value arrays are calculated for each candidate allocation
- **Branch:** create *b* hypotheses for each candidate allocation

Procedure for MMR

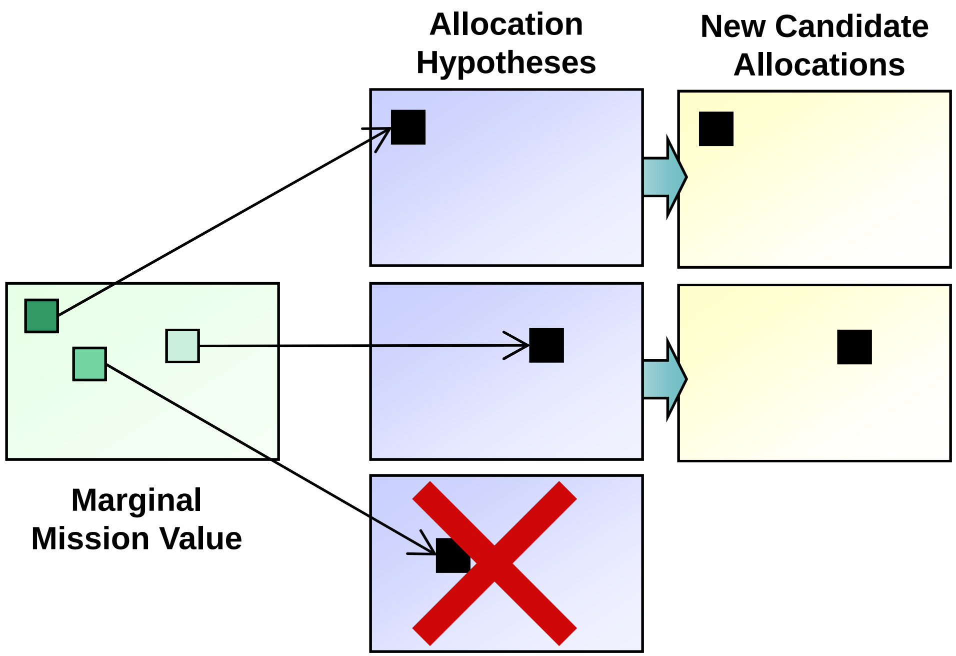




MMR

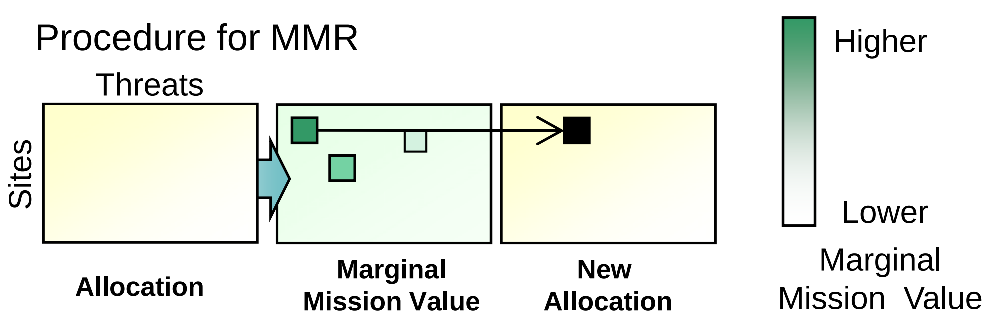
- One Marginal Mission Value array is calculated
- Assign interceptor to site/ threat combo that maximizes marginal MV

MHMMR - Prune



MHMMR

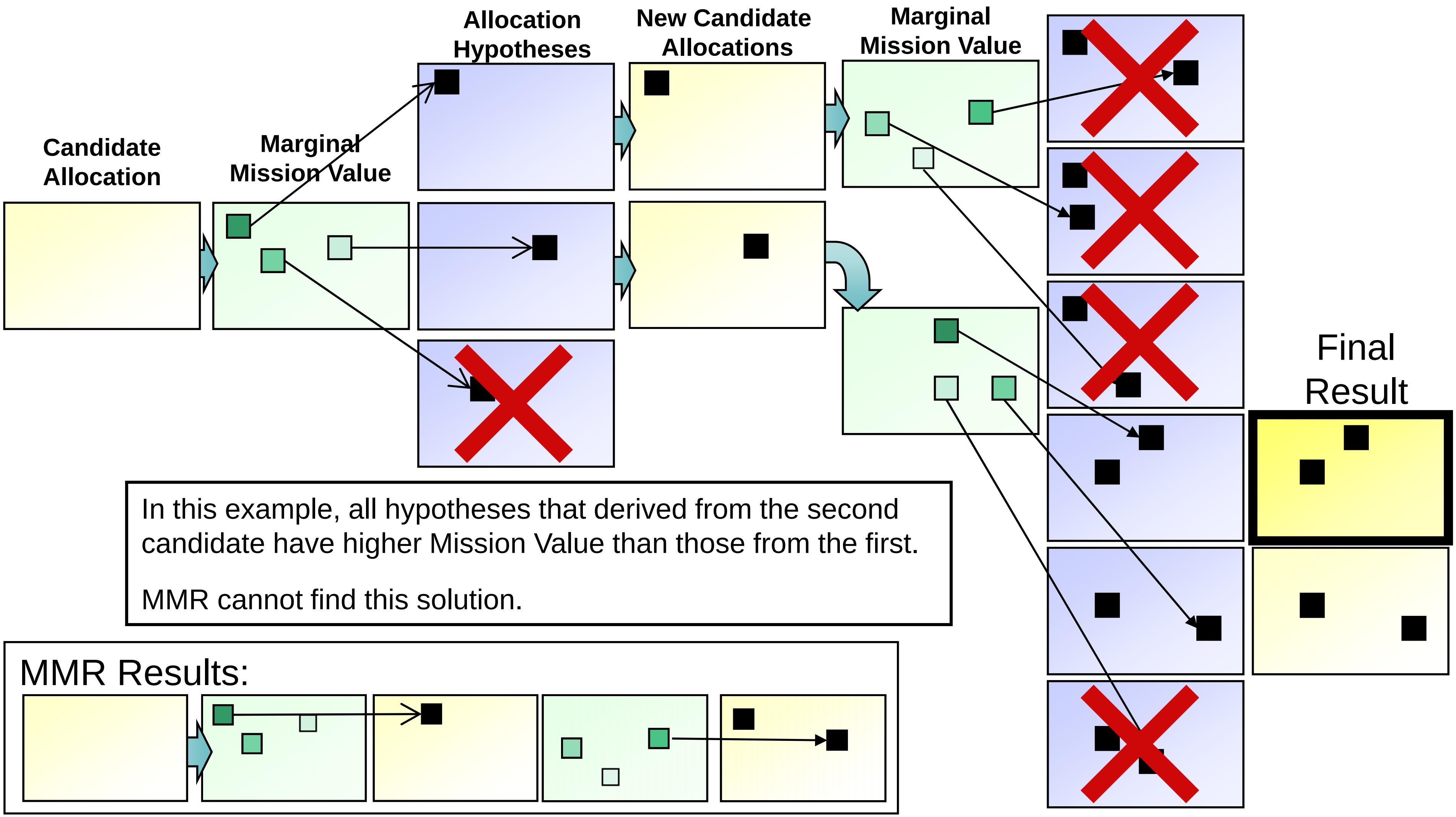
- **Prune:** maintain the *p* best hypotheses as candidate allocations



MMR

- One Marginal Mission Value array is calculated
- Assign interceptor to site/ threat combo that maximizes marginal MV

MHMMR – Branch and Prune



In this example, all hypotheses that derived from the second candidate have higher Mission Value than those from the first.

MMR cannot find this solution.

MMR Results:

MHMMR – Conclusions

- Implementation details
  - Branching needs to be independent of specific mission value function used. Maintain modularity.
  - Pruning needs to give distinct candidate allocations. If same hypothesis arises from two different candidates, include it at most once in next set of candidates.
- Branch/Prune parameters
  - Since all marginal mission values are calculated to determine branching, low additional computation cost required to maintain large number of branches.
  - Using only 1 branch reduces MHMMR to classic MMR. Since MHMMR contains MMR, MHMMR is guaranteed to outperform (or at least equal) MMR.
- Quick to implement

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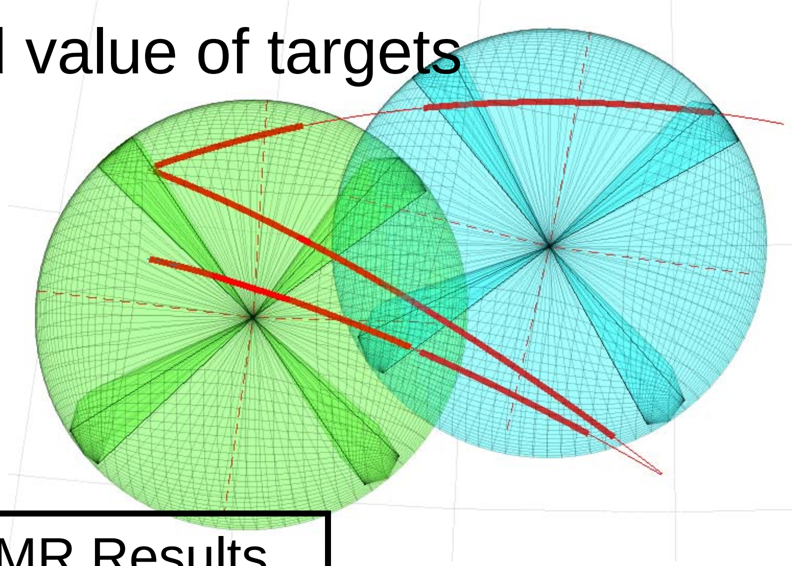
Target-Based Mission Value

- Goal: Maximize the total expected value of targets (incoming threats) destroyed
- 3 targets with values 10, 10 and 40
- 2 sites, both with 3 interceptors.
- pK = [.99 .9 .5
  .5 .01 .4]



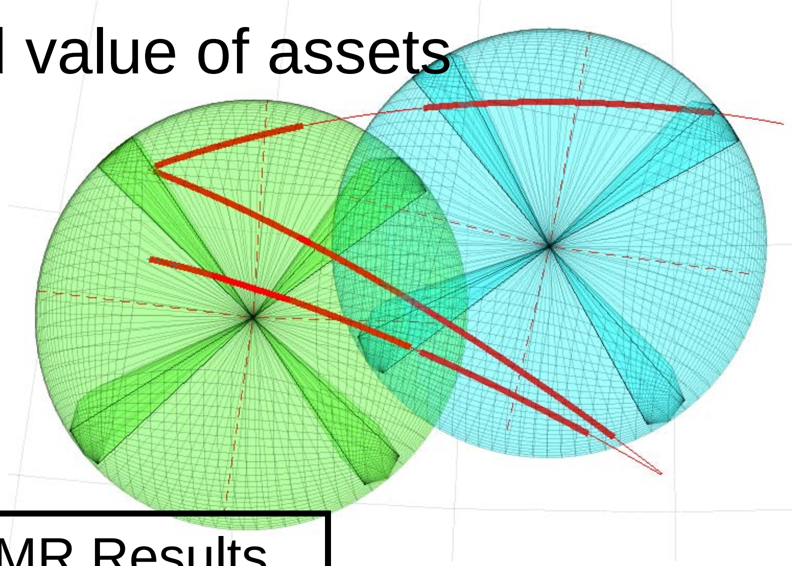
	MMR Results	MHMMR Results
Allocation	1 0 2 0 0 3	1 1 1 0 0 3
Mission Value	47.74	54.58 (optimal!)
Run-time	0.01 sec	0.015 sec

Mission Value Function

$$\sum_{j=1}^{n} V_j \left[1 - \prod_{i=1}^{m} \left(1 - p_{ij}\right)^{x_{ij}} \right]$$

Asset-Based Mission Value

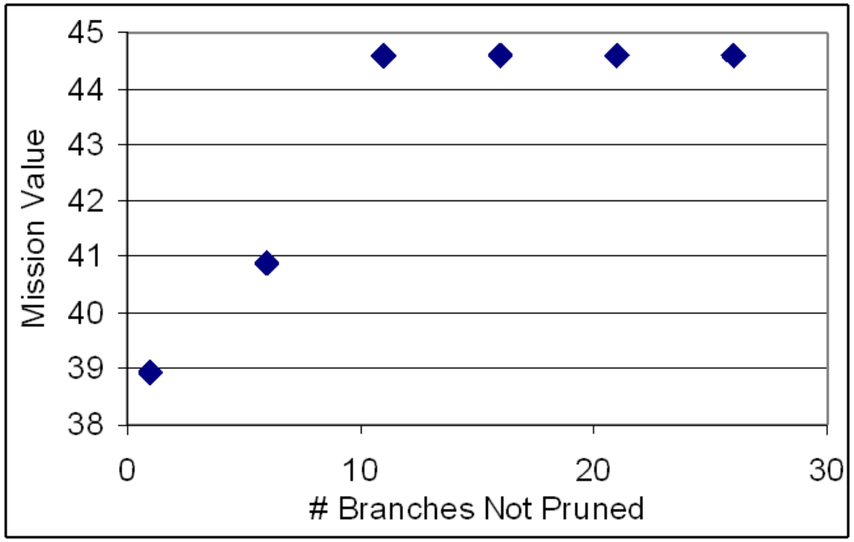
- Goal: Maximize the total expected value of assets protected
- 3 targets with values 10, 10 and 40
- 2 sites, both with 3 interceptors.
- pK = [.99 .9 .5
  .5 .01 .4]



	MMR Results	MHMMR Results
Allocation	0 0 3 0 0 3	1 1 1 0 0 3
Mission Value	38.92	44.59 (optimal!)
Run-time	0.01 sec	0.05 sec

MHMMR Performance

- Adjusting branching and pruning parameters changes behavior:
  - If one branch is kept and one branch is created, MHMMR is equivalent to MMR
  - If no branches are pruned and all branches are created, MHMMR is equiv. to enumeration
- Increasing the number of branches kept increases expected mission value
- Increasing the number of branches may increase expected mission value



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Conclusions

- MHMMR is a fast algorithm that is appropriate for solving the WTA problem for a wide variety of MV functions
  - Demonstrated optimal performance on specific scenario with target-based and asset-based MV.
- It may be possible to develop better approaches for any particular MV function, but MHMMR provides a quick check that is better than the classic MMR
  - Classic MMR is unable to solve asset-based MV. MHMMR could with no modifications to underlying algorithm
- Does not require much labor to implement; low risk development